1. This problem deals with rationality.
(a) (12 points) Of the following statements, identify all that are the expected functions of a market:
A. Potentially redistributes resources among buyers and sellers
B. To avoid confusion, ensures that a sell bid exists before taking any buy bids
C. To prevent unfairness, ensures that successive buy bids cannot go down (cannot reduce prices) and successive sell bids cannot go up (cannot increase prices)
D. Ensures that a deal takes place between some buyer and some seller
Solution: A -3 per wrongly checked or wrongly unchecked choice
(b) (5 points) A discriminatory price auction
A. Is unconstitutional in most modern democracies
B. Avoids the impossibility result of Meyerson \& Satterthwaite, which affects uniform price auctions
C. Is commonly used wherever people stand in line to buy tickets, because those who show up first get priority over those who show up later
D. Is based on the idea of setting the price differently for each allowed trade

## Solution: D

(c) (8 points) Consider an auction scheme where a trade takes place between a seller and a buyer only if the given buyer bids strictly higher than what the given seller bids. Moreover, the price the buyer pays the seller equals the geometric mean of their respective bids. [The geometric mean of two positive real numbers $x$ and $y$ is $\sqrt{x \times y}$.] Of the following statements, identify all that are true.
A. This auction is budget balanced
B. This auction is incentive compatible for buyers
C. This auction is incentive compatible for sellers
D. Assuming that sellers and buyers bid according to their true valuations, this auction yields a Pareto optimal allocation of resources

## Solution: A

-2 per wrongly checked or wrongly unchecked choice
D is false because if there are two buyers and one seller and both buyers bid higher than the seller, the lower of the two buyers may win
(d) An auction house has received sealed bids in order $A_{0}, A_{1}, \ldots, A_{9}$ as shown below:

| Amount | Sell Bids | Buy Bids |
| :--- | :--- | :--- |
| $\$ 9$ |  | buy $A_{2}, A_{8}$ |
| $\$ 8$ | sell $A_{5}$ |  |
| $\$ 7$ |  |  |
| $\$ 6$ | sell $A_{7}$ | buy $A_{6}$ |
| $\$ 5$ |  | buy $A_{9}$ |
| $\$ 4$ |  |  |
| $\$ 3$ | sell $A_{0}, A_{3}$ | buy $A_{1}, A_{4}$ |
| $\$ 2$ |  |  |
| $\$ 1$ |  |  |

i. (5 points) The price computed under the $\mathrm{M}^{\text {th }}$-price auction is
A. $\$ 7$
B. $\$ 6$
C. $\$ 5.50$
D. None: there is no deal

## Solution: B

ii. (5 points) The price computed under the dual-price auction is
A. $\$ 7$
B. $\$ 6$
C. $\$ 5.50$
D. None: there is no deal

## Solution: B

iii. (5 points) Under the dual-price auction,
A. $A_{0}, A_{3}, A_{7}$ sell to $A_{2}, A_{6}, A_{8}$
B. No one sells to $A_{8}$, because its bid is the last of the eligible bids
C. $A_{0}, A_{3}$ sell to $A_{2}, A_{8}$
D. $A_{7}$ sells to $A_{6}$, because their prices match and they are right in the middle

## Solution: C

