Name: $\qquad$

## Homework 4 for CSC 513: E-Commerce Technologies

## Collaborative Work

You may form teams of 1-4 members (of students in this class) to cooperate on this problem set. After discussing the problem, please write up your answers individually. Indicate the names of the other members in your team, if any.

## 1 CBMG

|  | $\mathbf{E}$ | $\mathbf{S}$ | $\mathbf{P}$ | $\mathbf{X}$ |
| :--- | :--- | :--- | :--- | :--- |
| E |  | 0.8 | 0.1 | 0.1 |
| S |  | 0.5 | 0.4 | 0.1 |
| P |  | 0.5 | 0.2 | 0.3 |
| X |  |  |  |  |

1.1. (20 points) The above matrix describes a CBMG involving the states enter ( E ), search (S), pay (P), and exit (X). Compute how many times (on average) each state is visited during each session.

## 2 Performance

2.1. ( 30 points) We are given an e-commerce site with one function of interest: search. The following components are of interest: application server processor (A), application server disk (D), web server processor (W), and network uplink (U).

We conduct a load test involving only search. From this test, we learn the following.

- The site can support 10,000 search requests in 10 minutes with the utilizations for $\mathrm{A}, \mathrm{D}, \mathrm{W}$, and U at $20 \%, 50 \%, 20 \%$, and $30 \%$, respectively.

Determine the following.

- Service demand for each component for the given test.
- Maximum throughput of the system.
- Maximum throughput of the system after the capacity of D is doubled.


## 3 Rationality

3.1. (20 points) Consider the following lottery. You are given two shuffled decks ( L and R ) of cards numbered 1 to 10 . You draw a card from each deck. If the L card equals the R card, you are paid $\$ 10$. If the L card is smaller than the R card, you replace both cards, shuffle the decks, and draw again. Otherwise, the game end. Using high-school algebra, compute the expected payoff of this lottery.
3.2. (20 points) Describe Pareto Optimality in about 50 words.

## 4 Auctions

4.1. (30 points) An auction house has received sealed bids in order $A_{0}, A_{1}, \ldots, A_{9}$. Compute the prices for Mth, $(\mathrm{M}+1)$ st, and dual price auctions. Also list the parties who end up selling and those who end up buying under these auctions (ensure efficiency and break any ties in favor of the earlier bidders).

| Amount | Sell Bids | Buy Bids |
| :--- | :--- | :--- |
| $\$ 9$ |  | buy $A_{2}$ |
| $\$ 8$ | sell $A_{5}$ |  |
| $\$ 7$ | sell $A_{1}$ | buy $A_{4}$ |
| $\$ 6$ |  |  |
| $\$ 5$ |  | buy $A_{8}$ |
| $\$ 4$ |  |  |
| $\$ 3$ | sell $A_{0}, A_{3}$ | buy $A_{6}, A_{7}, A_{9}$ |
| $\$ 2$ |  |  |
| $\$ 1$ |  |  |

Name: $\qquad$

## Homework 4 for CSC 513: E-Commerce Technologies

## Independent Work

You must solve this problem set individually without any assistance from anyone.

## 5 CBMG

5.1. (20 points) Consider a simple e-commerce site with the following states: enter $(\mathrm{E})$, search $(\mathrm{S})$, pay $(\mathrm{P})$, and exit (X). The following matrix describes the CBMG for this site (the blank entries are 0). The start of a session is given by E and the end of a session by X .
Notice that there are cycles in this CBMG. Consequently, some of the states may be entered more than once within a given session.
Express the expected number of times a given state is entered (per session) with the lowercase version of its name, i.e., e, s, p, and $x$. That is, e refers to the number of times that state E will be visited per session, s refers to the number of times that state $S$ will be visited per session, and so on. Based on the above matrix, formulate linear equations for these variables. Solve the equations for the four variables.

|  | $\mathbf{E}$ | $\mathbf{S}$ | $\mathbf{P}$ | $\mathbf{X}$ |
| :--- | :--- | :--- | :--- | :--- |
| E |  | 0.9 |  | 0.1 |
| S |  | 0.5 | 0.2 | 0.3 |
| P |  | 0.5 |  | 0.5 |
| X |  |  |  |  |

## 6 Performance

6.1. ( 30 points) We are given an e-commerce site with two functions of interest: browse and funds transfer. The following components are of interest: application server processor (A), application server disk (D), web server processor (W), and network uplink (U).
We conduct two separate load tests, one involving only browse and the other involving only funds transfer. From these tests, we learn the following.

- The site can support 10,000 browse requests in 10 minutes with the utilizations for $\mathrm{A}, \mathrm{D}, \mathrm{W}$, and $U$ at $20 \%, 50 \%, 20 \%$, and $40 \%$, respectively.
- The site can support 1,000 funds transfers in 10 minutes with the utilizations for $\mathrm{A}, \mathrm{D}, \mathrm{W}$, and U at $60 \%, 20 \%, 10 \%$, and $10 \%$, respectively.

Determine the following.

- Service demands for each components for the given tests.
- Maximum throughput of the system separately in terms of browse and funds transfer requests.


## 7 Auctions

7.1. ( 30 points) An auction house has received sealed bids in order $A_{0}, A_{1}, \ldots, A_{9}$. Compute the prices for Mth, $(\mathrm{M}+1)$ st, and dual price auctions. Also list the parties who end up selling and those who end up buying under these auctions (ensure efficiency and break any ties in favor of the earlier bidders).

| Amount | Sell Bids | Buy Bids |
| :--- | :--- | :--- |
| $\$ 9$ |  |  |
| $\$ 8$ |  | buy $A_{6}$ |
| $\$ 7$ |  | buy $A_{4}, A_{9}$ |
| $\$ 6$ | sell $A_{7}$ |  |
| $\$ 5$ |  | buy $A_{8}$ |
| $\$ 4$ | sell $A_{5}$ |  |
| $\$ 3$ | sell $A_{0}, A_{3}$ | buy $A_{2}$ |
| $\$ 2$ | sell $A_{1}$ |  |
| $\$ 1$ |  |  |

