Non-Monotonic Logic and System G[1]

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Abstract

This note refutes the conjecture that Boolos' modal logic **G** *forms a basis for an interesting non-monotonic logic.*

In [2] and [3] I conjectured that the correct non-monotonic logic is the weak logic of [3] strengthened by axiom schema corresponding to the theorems of Boolos' modal logic **G** [1]. The motivation for this conjecture was the view that since nonmonotonic logic is a logic concerning provability, it ought to reduce to the logic of ordinary provability "in the limit" as the specifically nonmonotonic aspects were ignored. Because **G** captures the logic of provability in Peano arithmetic, I concluded it was the correct "limit" for such reductions. I was wrong.

The interesting property of **G** is that it exactly captures the notion of provability in Peano arithmetic (**PA**) by means of its modality. Put another way, **G** exactly characterizes the set of theorems of the axiom set **PA** in the set of all formulas L. **G** exactly describes the sets $Th(\mathbf{PA})$ and $L - Th(\mathbf{PA})$.

The point of non-monotonic logic, however, is to describe the notion of provability under varying sets of axioms and assumptions. For each set of axioms and assumptions A, the modality of non-monotonic logic should characterize the sets of formulas $Th(A)$ and $L - Th(A)$. Since **PA** is merely one possible set A, restricting the modality of non-monotonic logic to describing $Th(\mathbf{PA})$ and $L - Th(\mathbf{PA})$ misses the point of the logic.

The weak logic of [3] does require strengthening to capture a better notion of provability, but should not include notions of provability from a fixed set of axioms.

1. Boolos, G., 1979. *The Unprovability of Consistency*, Cambridge: Cambridge University Press.

2. Panel on non-monotonic logic, *First Annual Conference of the American Association for Artificial Intelligence*, Stanford, California, August 1980.

3. McDermott, D., and Doyle, J., 1980. Non-monotonic logic—I, *Artificial Intelligence* **13**, 41-72.

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