A Rational Mechanics of Reasoning

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Abstract

When artificial intelligence seeks to make reasoning "mechanical," it means reasoning carried out by a machine or definite procedure rather than a form of behavior satisfying the laws of mechanics. This paper describes work on the mechanization of reasoning in the latter sense, aiming to formalize reasoning systems using the science of mechanics. The paper describes how the formal concepts of modern axiomatic rational mechanics apply directly and with slight adaptations to psychological systems as well to the familiar physical applications. These nonphysical applications provide a new formal vocabulary for characterizing realistic notions of rationality and limits on reasoning abilities.

In memoriam
Clifford Ambrose Truesdell III (1919–2000)
Herbert Alexander Simon (1916–2001)

1 Introduction

Over three and a half centuries ago Descartes promulgated a dualistic theory of mind in which a mental substance of the mind accompanied the physical substance of the body. His theory fit well with the discourse of the time, which spoke of minds and bodies acted upon and moving in response to forces, just as it does today. Despite this mechanical cast and the great advances in mechanics later achieved by Newton, Euler, and others, the Cartesian view declined in both natural and mental philosophy. In part this happened because Descartes' mental substances lacked position, meaning that mental actions lacked description in terms of the physical motion treated by mechanics. The new mechanics thus offered no way to apply its developing formal concepts to understanding the relation of minds to the body or the nature of forces acting on minds.

The failure of mechanics to treat the mind as well as the body disadvantaged the mental sciences relative to the physical. Most notably, this failure denied psychology concepts and methods central to the success of mathematical mechanics in transforming science and technology. In mechanics, one characterizes different materials by the forces they generate, by their response to forces and motion, and by the limits their mass and these forces place on their motion, but no psychological theory allows a corresponding methodology of specification and analysis. Apart from a few isolated explicitly mechanical discussions (e.g., (Herbart 1891; Shand 1920)), psychology and economics have made do with informal notions of motivation and habit, with formal notions of logic, probability, utility and rationality, and with developing notions of limited rationality. These theories provide valuable insight and understanding, but fail to provide some of the simplest concepts offered by mechanics, such as the notions of mass and inertia that even today provide the commonsense basis for many truisms about human behavior. Everybody knows, for example, that you have to force yourself to change; that the greater the desired change, the more you have to work at it; that the more you know, the harder it is to change your position; and that to maintain direction or focus you have to force yourself to ignore distractions.

This paper indicates how recent advances in mechanics provide formal concepts of force, mass, and momentum that enable one to transform some heretofore metaphorical mental applications of these terms into meaningful, true or false, non-metaphorical statements within the axiomatic framework of modern rational (i.e., conceptual and mathematical) mechanics (Truesdell 1991). This augmentation of the existing technical conceptions of logic, economics, and computational intelligence with the formal concepts of mechanics permits construction of mechanical theories of the interaction of mind and body, limits on ideal economic rationality, and the character of everyday psychology.

More specifically, identifying mechanical forces and masses in psychology and economics provides a new formal vocabulary for characterizing limits on rationality. This new vocabulary uses mass and force to express limits on the speed with which agents can change mental state and direction in accomodating new information and in reasoning and deliberation. It analyzes difficulties in maintaining focus of attention in terms of the magnitude and direction of forces resulting from superposition of competing motivations, and in terms of the work required to counteract distracting forces. It views some equilibrium notions of economics in terms of static balance of forces and relaxed or equilibrium states of materials. Mechanical limitations on the rapidity and effort of change and the maintainance of attention do not account for all limitations on rationality, but they represent some of the most important limitations that current psychological theories characterize poorly.

The short summary that follows cannot present the axioms, applications, or implications in detail, but aims instead to sketch some key concepts and structures used in the theory. The following first summarizes the structure of modern rational mechanics and the peripheral deformations employed to bring the mind into the mechanical fold. The presentation then describes the character of mechanical axioms and the mathematical structures to which they give rise, how these structures follow standard mechanical formalism and where they depart. Following this, two vignettes illustrate the application of mechanics to psychological and artificial reasoning systems.

2 Rational mechanics

Modern theories of mechanics owe much of their shape to the axiomatic and mathematical rational mechanics of Walter Noll (1958; 1963; 1964; 1973; 1972) and Clifford Truesdell (1965; 1991), who along with others reinvigorated mechanics half a century ago by clarifying and strengthening the foundations with modern mathematical concepts. Most notably, Noll's axioms moved beyond the traditional Newton-Euler formulations of mechanics to provide the first axiomatic characterization of the concept of force.

These modern mechanical theories regard bodies as subject to general laws applying to all types of materials, laws that characterize geometry of space, time and motion, the structure of material bodies, the nature of systems of forces and the relation of force to motion. None of the most general laws say anything about which forces exist, or even that *any* particular forces exist. Such statements instead come in special laws of dynamogenesis that characterize the origin of forces. Other special laws characterize the behavior of special types of materials, ordinarily identified in terms of constraints on bodies, configurations, motions, and forces. For example, rigid body mechanics comes from adding to the general theory kinematical constraints that fix the relative distances of body parts, while the theory of rubber comes from augmenting the general theory with the configuration-dependent forces characteristic of rubber. Mechanics uses the term *constitutive assumption* to refer to the special laws for particular materials, since the laws reflect assumptions about the constitution of the material. Mechanical practice depends critically on these special laws.

Physical materials exhibit a diversity broader than many people realize. This diversity requires the general laws to characterize the structure of bodies, forces, and motions at a level separate from the usual properties familiar from elementary textbooks. This separation provides most of the mathematical generality needed to cover mental bodies, forces, and motions in addition to familiar physical ones. The difference between mind and body proves not terribly greater than the differences already existing among physical materials and forms of motion. The key difference arises in the structure of the spaces inhabited by these materials. By seeking to understand physical reality itself rather than continuing assumptions made for increasingly-dated mathematical convenience, Noll obtained axioms that distinguish central mechanical concepts from many special characteristics of physical space and time. Turing (1936) and others developed complementary theories of computation and reasoning providing examples critical to finding mathematical spaces that characterize minds. The present development shows how to find common ground in structures for space, time, force and materials that both serve mechanical and mental needs.

3 Departures

We broaden mechanics by departing from the axioms formulated by Noll and Truesdell in two ways. The first departure develops the notion of *hybrid* mechanical systems, in which disparate types of space, matter, and force can interact. The second departure characterizes *discrete* mechanical systems that augment ordinary temporal and spatial continua with discrete spaces and motions.

Our departures here compare with recent work on so-called hybrid system models for describing systems exhibiting both discrete and continuous behaviors (Alur *et al.* 1993; Branicky 1995; Davoren 1998). The primary differences lie in treating all factor spaces as linear mechanical spaces, as opposed to mere continuous or discrete spaces, and in extending the hybrid structure to include a hybrid mechanical dynamics, as opposed to a mere hybrid kinematics and nonmechanical equations of motion. Development of discrete mechanical dynamics also distinguishes the constructions here from the discrete Lagrangian dynamics of Baez and Gilliam (1994), which, as with the Lagrangian approach in general, embodies no mechanical notions but instead requires choosing a Lagrangian function that encodes all the desired mechanical laws (Sussman, Wisdom, & Mayer 2001).

The mechanical axioms obtained in this way do not in themselves say how to view minds as mechanical systems, any more than they prescribe how to view the human body as a mechanical system, but they open the possibility that human minds and bodies together constitute a hybrid mechanical system with both discrete and continuous elements.

4 Events, Time, and Space

Noll's first set of axioms characterizes *neo-classical event worlds* of time, space, duration and distance in terms of a set $\mathcal{W}$ of *events*, a *time-lapse* metric $\hat{t} : \mathcal{W} \times \mathcal{W} \to \mathbb{R}$ giving the length or duration of the temporal interval between events, and a *distance* metric $\hat{d} : \Sigma \to \mathbb{R}^+$ on simultaneous events giving the distance between them. The axioms relating these concepts yield a translation space over simultaneous events taking the familiar form of a three-dimensional Euclidean inner-product vector space. Although Noll's axioms require that times and the space of locations forms continua, he notes that most of his development of space works just as well for many discrete spaces.

We first vary Noll's axioms to assume that the set of events comprises several factor event worlds described by their own time-lapse and distance metrics, so that each hybrid event plays distinct roles in each factor system. Each hybrid event must correspond to a unique combination of factor events. The factor temporal orderings of events must admit a common refinement, but otherwise the temporal metrics need not bear any relation to each other, and the sets of instants need not form continua. For simplicity we require that the sets of events at each instant form isomorphic spaces of locations, but the factor spaces of locations need have no special metric or other relation to each other. Our axioms do not require more than one factor system. One need only assume that one factor system exists and exhibits the standard three-dimensional continuum structure to obtain the event world of traditional mechanics.

We next vary Noll's axioms to avoid the general assumption of spatial continua. We follow his development within each factor space, requiring that each factor space of locations admit a translation space having the structure of an inner-product vector space that gives each isometry (distance-preserving, rigid mapping) the structure of a composition of a translation and orthogonal transformation (rotation), but allow finite fields of scalars as well as the usual continuum fields. We obtain the hybrid space of locations as the product of the factor spaces, and the hybrid translation space $\mathcal{V}$ as the product $\prod_i \mathcal{V}_i$ of the factor translation spaces. The hybrid translation space, however, can differ in one important way from the factor translation spaces. When the factor vector spaces employ different fields of scalars (up to isomorphism), the scalars for the product form a ring but not a field. The hybrid translation space thus in general forms a module rather than a vector space (cf. (Kalman, Falb, & Arbib 1969)), an algebraic structure that provides almost all the linear structure of vector spaces, but not necessarily a unique notion of dimension.

5 Bodies

Noll's second set of axioms characterizes *material universes* of bodies and their parts. A material universe consists of a set Υ of *bodies* together with a *subbody* or part-of relation $\preceq$ on $\Upsilon \times \Upsilon$. His axioms imply that the material universe forms a boolean lattice in which each body has $\mathcal{B} \in \Upsilon$ a unique exterior $\mathcal{B}^e$. We follow Noll and almost all treatments of mechanics and assume for simplicity that each body consists of a set of *body points*. Bodies consisting of single points can represent the point-masses of analytical mechanics, while bodies consisting of continua of points of different dimensions represent the solids, shells, rods, and other bodies studied in continuum mechanics.

Hybrid mechanics permits disparate factor material universes, so that not every body in one factor mechanics need correspond to any particular body in another factor. The material universe of Cartesian minds, for example, need not contain elements corresponding to tables, typewriters, or other purely physical bodies. We obtain the hybrid material universe as the lattice product (or equivalently, lattice sum) of the factor material universes.

6 Motion

We follow Noll in separating the notions of configuration and deformation of a body from the notions of placement in space. The former notions concern changes of the spatial interrelations of body elements, and thus what one can think of as actual motion, while the latter notion reflects the framing of locations from the perspective of an observer, which one can think of as mere apparent motion. As usual, we identify the notion of an observer or frame of reference with a time-indexed family of rigid transformations.

Standard mechanics assumes continuity of the first two derivatives of motions and frames of reference. Such derivatives need not exist for discrete conceptions of time, nor for motion in discrete spaces. Various approaches seek to compensate for lack of continuity concepts. One of these, the smoothest-path principles discussed by Truesdell (1984), exhibits connections with minimum-change principles in reasoning.

7 Forces

Noll's third set of axioms characterizes the forces existing at each instant in terms of *force systems*. A force system consists of a mapping $f : \Upsilon \times \Upsilon \to \mathcal{V}$ of pairs of bodies to spatial vectors, where $f(\mathcal{B}, \mathcal{C})$ denotes the force exerted on $\mathcal{B}$ by $\mathcal{C}$ in the system of forces f. The main axioms for forces state that forces exerted on or by separate subbodies of a body combine additively, and that the sum of all forces on each body vanishes. This last condition may seem strange, but really just amounts to treating all components of familiar force equations impartially. In particular, this axiom leads to treating the inertial force $-ma$ as just one special sort of force cancelling out the sum of all the other forces on the body, thus rewriting the familiar $f = ma$ as $f - ma = 0$. Each force system induces two subsidiary force systems, internal forces between separate parts of a body, and external forces exerted by the exterior of a body on its parts. Noll's axioms also state that the internal forces are contact forces that vary continuously with the area of the contact boundary between the two parts, and that the external forces can be by contact and at a distance and vary continuously with the mass, volume and contact boundary areas of the parts.

These same axioms apply essentially without change in the hybrid mechanical setting, implying additivity, balance, and decomposition of forces in each factor mechanics. We depart from the usual axioms in separating out the continuity assumptions, as these do not apply to discrete factor systems. We also separate out Noll's torque axioms because the discrete factor mechanics can involve spaces with more than three dimensions while the ordinary definition of torque requires space having at most three dimensions.

Noll's also states three additional axioms restricting dynamogenesis. The first of these axioms states the principle of determinism, that the history of body and contact forces at preceding instants determines a unique value for these forces at a given instant. The second axiom states the principle of locality, that the forces at a given point depend only on the configuration of bodies within arbitrarily small neighborhoods of the point. The third axiom states the principle of frame indifference, that forces depend only on the intrinsic properties of motions and deformation, not on properties that vary with the reference frame. The axiom of frame indifference carries over directly to our mechanics, but we require the axioms of determinism and locality only at the hybrid level, since forces at one factor location may vary and vary widely with the location in other factor spaces.

8 Mass

Inertial forces represent the most basic of the general constitutive assumptions concerning fundamental physical forces. The standard axioms treat mass as a measure $m : \Upsilon \to \mathbb{R}^{0+}$ assigning to each body its nonnegative inertial mass value, and from this construct the momentum p with respect to the velocity observed in a frame of reference. Truesdell (1991) formulates axioms for inertial forces that state the existence of inertial frames of reference in which a body has constant momentum during some interval if and only if the resultant force on the body vanishes, and the equality of the resultant force on a body in an inertial frame with the negative derivative of the momentum of the body. Combining these axioms with the general balance laws of forces and torques yields Euler's fundamental laws $f = \dot{p}$ (i.e., $f = ma$) and $F = \dot{M}$ stating the respective balances of linear and rotational momentum.

Noll regards the axioms of inertia as special laws of dynamogenesis rather than general ones in part because forces generated by masses play an important role in many, but not all, mechanical problems. This provides precedent for recognizing that physical mass that need not always play a role in mental motion, and that mental mass that need not always play a role in physical motion.

Hybrid mechanics generalizes the traditional conception of mass directly by regarding hybrid mass as the vector of mass values in each of the component systems. One can view this as extending the Newtonian conceptual distinction between inertial mass and gravitational mass. For discrete factor mechanics, we also assume that mass values can inhabit discrete vector spaces.

9 Interacting mind and body

We illustrate the interaction of mind and body in the language of mechanics by using two component worlds, one physical, one mental, to model the body and mind of a person we can call René. For the physical world we take the standard mechanical model, and identify the body of René as a body $\mathcal{B}_p$ existing within a universe Υ_p of physical bodies. Similarly, for the mental world we identify the mind of René as a body $\mathcal{B}_m$ existing within a separate universe Υ_m of mental bodies. Our simple illustration then uses the hybrid body $\mathcal{B}_R = \mathcal{B}_p + \mathcal{B}_m$ to model the person René, and the hybrid body $\top = \top_p + \top_m$ to model the universal body, using $\top_p$ and $\top_m$ to denote the maximal bodies in the factor universes.

Let us now consider the total force acting on René at some instant. In the hybrid mechanical formalism, we write this force as $f(\mathcal{B}_R, \top)$. The separation of the mental and physical bodies and the axioms for forces then let us rewrite this force as

$$f(\mathcal{B}_R, \top) = f(\mathcal{B}_p, \top) + f(\mathcal{B}_m, \top). \tag{1}$$

Let us look first at $f(\mathcal{B}_p, \top)$, which represents the force on the physical body. We decompose $\top$ into separate components

$$\top = \mathcal{B}_R + \mathcal{B}_R^e = \mathcal{B}_p + \mathcal{B}_m + \mathcal{B}_p^{pe} + \mathcal{B}_m^{me}, \tag{2}$$

where $\mathcal{B}_p^{pe}$ denotes the physical environment of the physical body obtained as the relative complement of $\mathcal{B}_p$ with respect to the greatest physical body $\top_p$, and where $\mathcal{B}_m^{me}$ denotes the mental environment of the mind obtained as the relative complement of $\mathcal{B}_m$ with respect to the greatest mental body $\top_m$. With this partition of the hybrid universe, we apply the axioms for forces to rewrite the force on the physical body as

$$f(\mathcal{B}_p, \top) = f(\mathcal{B}_p, \mathcal{B}_p) + f(\mathcal{B}_p, \mathcal{B}_m) + f(\mathcal{B}_p, \mathcal{B}_p^{pe}) + f(\mathcal{B}_p, \mathcal{B}_m^{me}). \tag{3}$$

This just says the total force on the physical body consists of the sum of the forces exerted on the body by the body itself, by the mind, by the physical environment, and by the mental environment. From a similar decomposition, we obtain the force on the mind as

$$f(\mathcal{B}_m, \top) = f(\mathcal{B}_m, \mathcal{B}_m) + f(\mathcal{B}_m, \mathcal{B}_p) + f(\mathcal{B}_m, \mathcal{B}_m^{me}) + f(\mathcal{B}_m, \mathcal{B}_p^{pe}). \tag{4}$$

The forces $f(\mathcal{B}_p, \top)$ and $f(\mathcal{B}_m, \top)$ constitute hybrid forces, containing components in both the physical and mental worlds. We can thus decompose them into components, for example, writing

$$f(\mathcal{B}_p, \top) = f_p(\mathcal{B}_p, \top) + f_m(\mathcal{B}_p, \top). \tag{5}$$

The deformations of the physical body and the mind depend only on these respective components in the hybrid mechanics, so motion of the physical body depends only on

$$f_p(\mathcal{B}_p, \top) = f_p(\mathcal{B}_p, \mathcal{B}_p) + f_p(\mathcal{B}_p, \mathcal{B}_m) + f_p(\mathcal{B}_p, \mathcal{B}_p^{pe}) + f_p(\mathcal{B}_p, \mathcal{B}_m^{me}). \tag{6}$$

Thus the physical motion stems from physical forces, but the origin of these forces might include mental bodies.

Many physical theories assume that self-forces vanish, that $f(\mathcal{B}, \mathcal{B}) = 0$ holds for each body $\mathcal{B}$. In seeking mechanical formalization of conscious agents, however, the assumption of vanishing self-forces seems undesirable. Self-conscious or self-directed action naturally suggests a mind that exerts nonzero forces on itself, though one can expect necessity of such forces in the formalization to depend on the details of the psychological organization. In particular, one might formalize a mind organized into competing sets of mental subagents (as in (Minsky 1986)) as involving only forces between separate mental components, so avoiding the need for nonzero self-forces.

10 Mechanical reasoning

A more involved illustration shows how to view reasoning systems related to Doyle's (1979) reason maintenance systems (RMSs). The following sketch draws on the formalization presented in (Doyle 1994). The full demonstration shows that some systems of this type satisfy the axioms of mechanics, and thus constitute mechanical systems.

In the mechanical formalization, we view a RMS and its environment as complementary bodies in the universe of bodies. We assume a set $\mathcal{D}$ representing all nodes and reasons available for use by the RMS. The RMS has both a set of base reasons and a set of derived conclusions represented by nodes. Each element in $\mathcal{D}$ can be interpreted as a policy, denoted $A \setminus\!\!\setminus B \Vdash C \setminus\!\!\setminus D$, for mutually disjoint sets $A, B, C, D \subseteq \mathcal{D}$, of adding a set C of conclusions and removing a set D of deleters when the set of derived conclusions contains a set A of antecedents and lacks a set B of qualifications. We take the set of possible *In*/*Out* labelings of nodes as the space of positions. This space forms a binary-valued vector space $\mathbb{D} = 2^{\mathcal{D}}$ over the field $\mathbb{Z}_2$. Elements of $\mathbb{D}$ thus also can represent changes to positions, with the element $x - y$ representing the change from position y to position x. We view the set of base reasons as the mass of the RMS, and use $\mathbb{D}$ again to denote the space of possible mass values.

Trajectories of the RMS consist of sequences of tripartite states composed of a node labeling, a set of instantaneous label changes, and a set of base reasons. We represent such trajectories as sequences of triples $(x, \dot{x}, m)$ of vectors in $\mathbb{D}$. By construction, the label-change vector $\dot{x}$ constitutes the velocity of the RMS, so these vector triples represent position, change, and mass vectors. We view the velocity and mass components of such triples as constituting the instantaneous momentum $p = (m, \dot{x})$ of the RMS.

The most direct interpretation of RMS operations in mechanical terms considers trajectories of states that include the intermediate states occupied during node relabeling as well as the equilibrium states visible to the RMS environment. Each step in this trajectory corresponds to modification of the partial labeling by application of one or more reasons. This fine-grained motion of the RMS then terminates in an equilibrium state representing a stable labeling of the nodes.

We think of external instructions to change the base reasons as constituting forces having only mass-change components, and interpretation of reasons by relabeling operations as providing forces with only spatial (accelerative) components. In other words, we decompose the force contributions from the RMS environment and the RMS itself into an applied mass force

$$f_t^{\mathbf{a}} = (\dot{m}_t, \mathbf{0}) \tag{7}$$

produced by the environment and a spatial self force

$$f_t^{\mathbf{s}} = (\mathbf{0}, \ddot{x}_t) \tag{8}$$

produced by the RMS itself, thus yielding a total force

$$\begin{aligned} f_t &= f_t^{\mathbf{a}} + f_t^{\mathbf{s}} && (9) \\ &= (\dot{m}_t, \ddot{x}_t). && (10) \end{aligned}$$

For illustration, consider application of a single reason

$$r = A_r \setminus\!\!\setminus B_r \Vdash C_r \setminus\!\!\setminus D_r \tag{11}$$

to a state σ_t, in which the reason generates a force from the state that then acts on the state to produce a new state exhibiting the appropriate velocity and position. We assume for simplicity that the reason generates only a spatial force

$$f_r(\sigma_t) = (\dot{m}_t, \ddot{x}_t) = (\mathbf{0}, \ddot{x}_t) \tag{12}$$

in which the mass flux vanishes. The RMS interprets r as a conditional prescription of node labels, in that validity of the antecedent portion $(A_r \setminus\!\!\setminus B_r)$ requires application of the consequent portion $(C_r \setminus\!\!\setminus D_r)$. In particular, the consequent portion represents a function from labelings to change vectors, in that if the current state contains the labeling x, the consequent $C_r \setminus\!\!\setminus D_r$ indicates a change vector given by $(C_r \setminus x) + (D_r \setminus \bar{x})$, where $\bar{x} = \mathbf{1} - x$ indicates the vector or set complementary to x. Specifically, we define the force generated by the RMS from r as providing the acceleration

$$\ddot{x}_t = \begin{cases} (C_r \setminus x_t) + (D_r \setminus \overline{x}_t) - \dot{x}_t & \text{if } A_r \subseteq x_t \subseteq \overline{B}_r \\ \dot{x}_t & \text{otherwise.} \end{cases} \tag{13}$$

This yields a velocity of

$$\dot{x}_{t+1} = \begin{cases} (C_r \setminus x_t) + (D_r \setminus \overline{x}_t) & \text{if } A_r \subseteq x_t \subseteq \overline{B}_r \\ \mathbf{0} & \text{otherwise,} \end{cases} \tag{14}$$

and so

$$x_{t+1} = \begin{cases} x_t + (C_r \setminus x_t) + (D_r \setminus \overline{x}_t) & \text{if } A_r \subseteq x_t \subseteq \overline{B}_r \\ x_t & \text{otherwise.} \end{cases} \tag{15}$$

Elsewhere we present a more detailed analysis in a range of more interesting mathematical and mechanical terms. One such analysis shows how to interpret a set of reasons in terms of a potential function that determines stress tensors at given mental locations, so that the resulting stress satisfies Cauchy's First Law, $\dot{p} = B + \mathrm{div}(T)$.

11 Mechanical limitations

Simon (1955) sought to move attention in economics away from an ideal rationality generally absent in human behavior to more realistic and limited conceptions of rationality. Mechanics provides a vocabulary for expressing and analyzing some such limitations. We note only a few.

Some limitations on rationality find expression in kinematic constitutive assumptions restricting the structure of bodies, akin to familiar rigid-body assumptions. These include assumptions about inherent inferential capabilities that ensure that the reasoner automatically performs certain classes of inference (e.g., "obvious" inferences), possibly yielding lower bounds on intelligence levels.

Mass itself constitutes a limitation on rationality. Ideal rationality allows bounded changes of information to cause arbitrarily large changes of position (e.g., "Everything you know is wrong."). The mass of a reasoner limits motion so that larger changes require larger forces.

As mentioned earlier, Noll's force axioms assume that contact forces vary continuously with the area of the contact boundary and so exhibit magnitude bounds proportional to the area of contact. One might view the noisy communications channels of Shannon's (Shannon 1948) first fundamental theorem about channel capacity as finite-area contact regions with areas measurable in bits, corresponding to the dimensionality of the space of forces communicable across the boundary.

12 Conclusion

Mechanics displays its strength in continuing applicability to new materials, especially to materials no one had thought of earlier. The preceding discussion sketched how mechanical concepts also apply to old materials few had thought of as mechanical. Just as mechanical concepts provide insight and power in understanding and exploiting newly discovered or constructed physical materials, we expect mechanical concepts to someday provide similar benefits regarding natural and newly-constructed mental materials.

Artificial intelligence today rests on essentially kinematic conceptions akin to the interlocking gears of clockworks: wonderful devices for certain purposes, but limited in ability. The static and kinetic balance of forces one considers in moving beyond the kinematic realm supports the broader spectrum of machines from jet aircraft to die casters to combine harvesters to skyscrapers. Expanding the concepts of artificial intelligence to employ these richer concepts of mechanics, which have proven so fruitful in physical technology, promises to similarly broaden the range of artificial mental mechanisms.

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