Modeling through Markov Chains : where is the Risk ?

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Introduction - Motivation

Advantages of Markovian models :

well known from the specialists…

Criticisms against Markovian models :

Level 1 : *“times durations are exponentially distributed while this is not the case in my real problem”*

Level 2 : *“In order to adjust my time duration distributions, I have to increase the state space of my model and this model becomes less and less tractable ”*

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Introduction - Motivation

But :

- Some steady state performance measures do not depends on the time duration distributions,

- Some other performance measures are not very sensitive on the time duration distributions. An Erlang-3 may give practically the same numerical results wrt performance measures as an Erlang-10.

While :
much larger errors may be done just because of a misunderstanding of the considered application under the modeling stage !

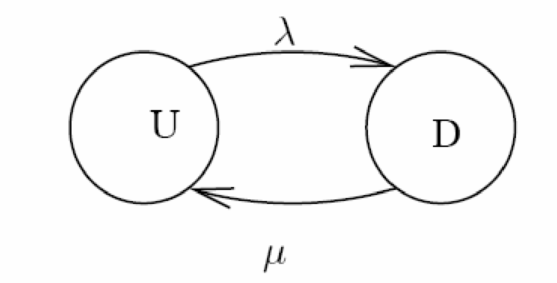
Introduction - Motivation

Here we want to illustrate this point on an example taken from the field of dependability.

To make things easy to understand, we chosen an simple example for which we were able to exhibit closed form expressions.

The non Redundant Element

Simplest model :



$$\overline{A} = \frac{\rho}{1+\rho} \qquad \rho = \lambda/\mu$$

If a spare part shortage may happen : probability α (from the field),

assume the spare parts arrive according to a Poisson Process w. rate γ

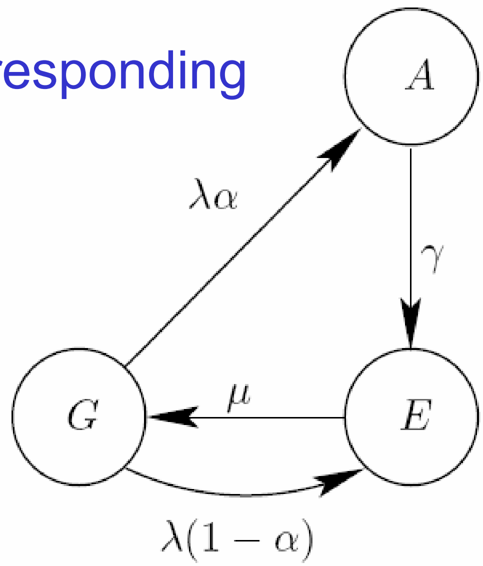
Heuristic of the practitioner :

$$\frac{1}{\hat{\mu}} = (1-\alpha)\frac{1}{\mu} + \alpha\left(\frac{1}{\gamma} + \frac{1}{\mu}\right) = \frac{1}{\mu}\left(1 + \frac{\alpha}{\phi}\right)$$

$$\overline{A} = \frac{\rho\left(1+\frac{\alpha}{\phi}\right)}{1+\rho\left(1+\frac{\alpha}{\phi}\right)} \qquad \phi = \alpha/\mu$$

The non Redundant Element

While looking at the Markovian model corresponding to the specifications we get



$$\pi_A = \frac{\alpha\rho}{\alpha\rho + \phi\rho + \phi} \qquad \pi_E = \frac{\phi\rho}{\alpha\rho + \phi\rho + \phi}$$

$$\pi_G = \frac{\phi}{\alpha\rho + \phi\rho + \phi}$$

$$\overline{A} = \pi_A + \pi_E = \frac{\rho\left(1 + \dfrac{\alpha}{\phi}\right)}{1 + \rho\left(1 + \dfrac{\alpha}{\phi}\right)}$$

Same expression as in the simplified model !

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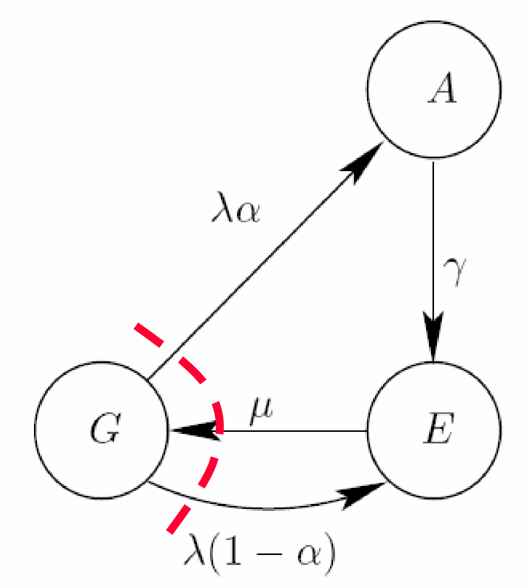
The non Redundant Element

More over :

$$\overline{A} = \frac{MDT}{MUT + MDT}$$

$$\lambda \pi_G = \frac{1}{MUT + MDT}$$

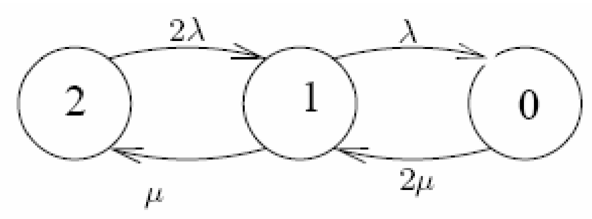
$$MDT = \frac{1 - \pi_G}{\lambda \pi_G} = \frac{1}{\mu}\left(1 + \frac{\alpha}{\phi}\right) = \frac{1}{\hat{\mu}}$$



as in the simplified model ! Heuristic +++ !

The Redundant Substructure

Without spare part shortage :



$$\overline{A} = \left(\frac{\rho}{1+\rho}\right)^2$$

(independence)

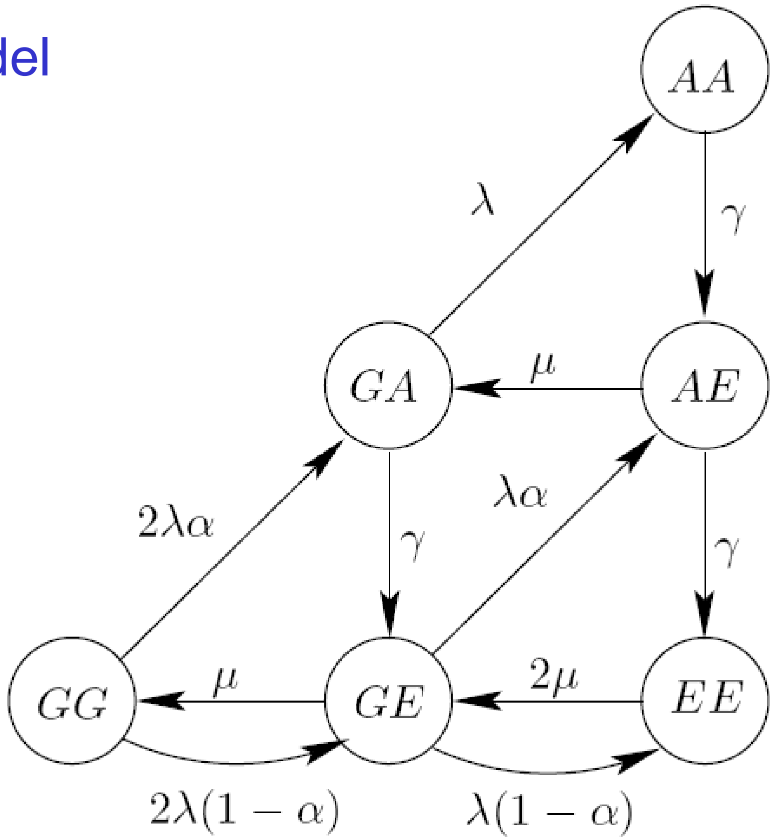
Using the heuristic with spare part shortage :

$$\overline{A} = \frac{\rho^2\left(1+\frac{\alpha}{\phi}\right)^2}{1+2\rho\left(1+\frac{\alpha}{\phi}\right)+\rho^2\left(1+\frac{\alpha}{\phi}\right)^2}$$

The Redundant Substructure

The more realistic Markovian model

GG : 2 good items
GE : exchanging 1 failed item
GA : waiting for 1 spare
AA : waiting for 2 spares
EE : exchanging 2 failed items
AE : waiting for 1 spare and
exchanging 1 failed item



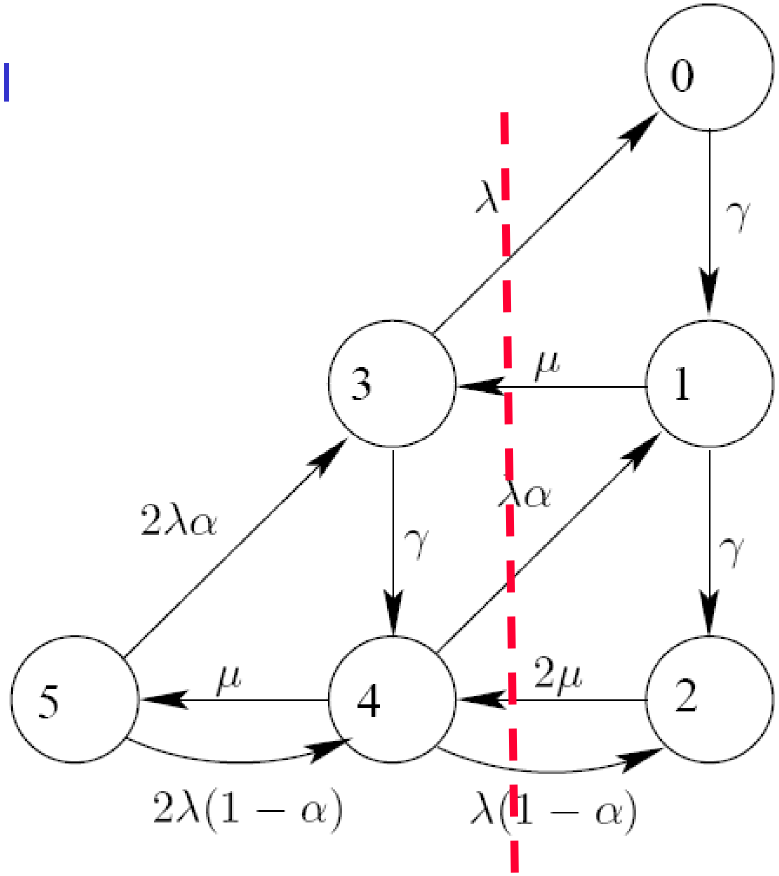
The Redundant Substructure

The more realistic Markovian model

$$\lambda(MUT + MDT) = \frac{1}{\pi_3 + \pi_4}$$

$$\lambda MUT = \frac{\pi_3 + \pi_4 + \pi_5}{\pi_3 + \pi_4}$$

$$\lambda MUT = 1 + \frac{1}{2\rho}\frac{\phi}{\alpha + \phi}$$
$$= 1 + \frac{1}{2\rho} \times \frac{1}{1 + \alpha/\phi}$$

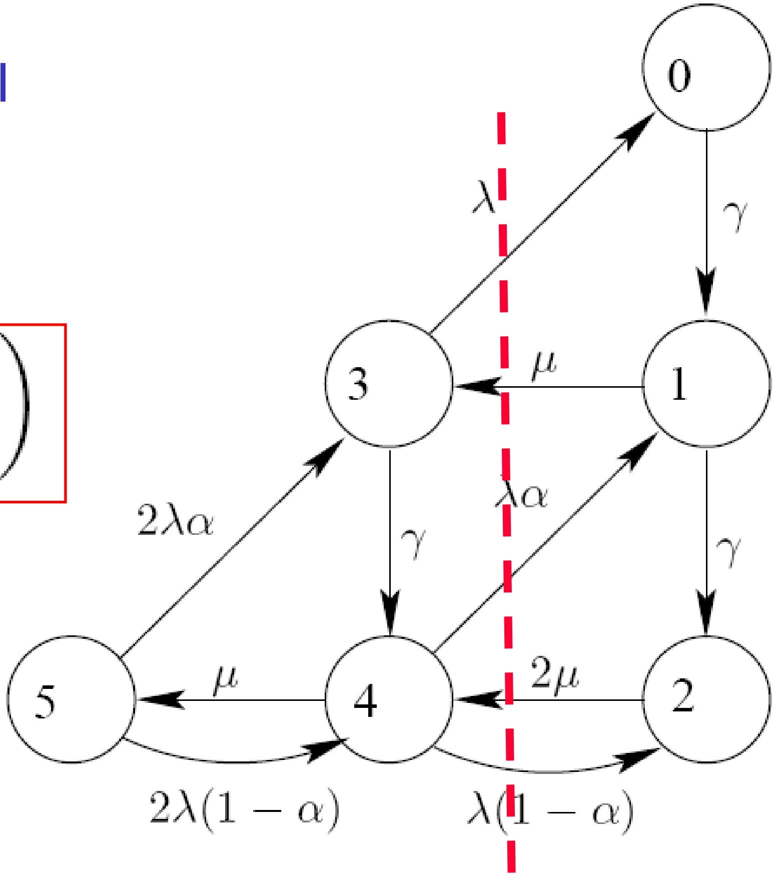


Same result as with the heuristic !!!

The Redundant Substructure

The more realistic Markovian model

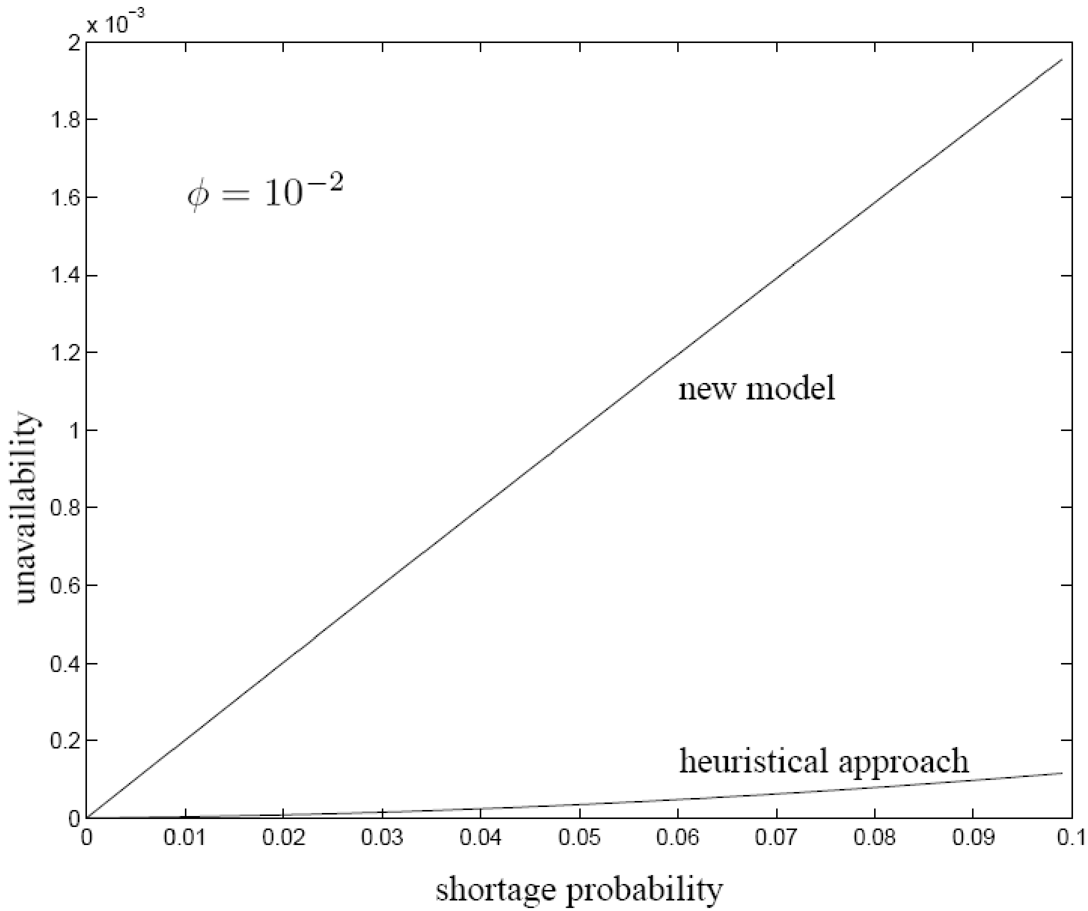
$$\boxed{2\mu MDT = 1 + \frac{\alpha}{\phi} \times \left(\frac{1+2/\phi}{1+\alpha/\phi}\right)}$$



Numerical Examples

Unavailability

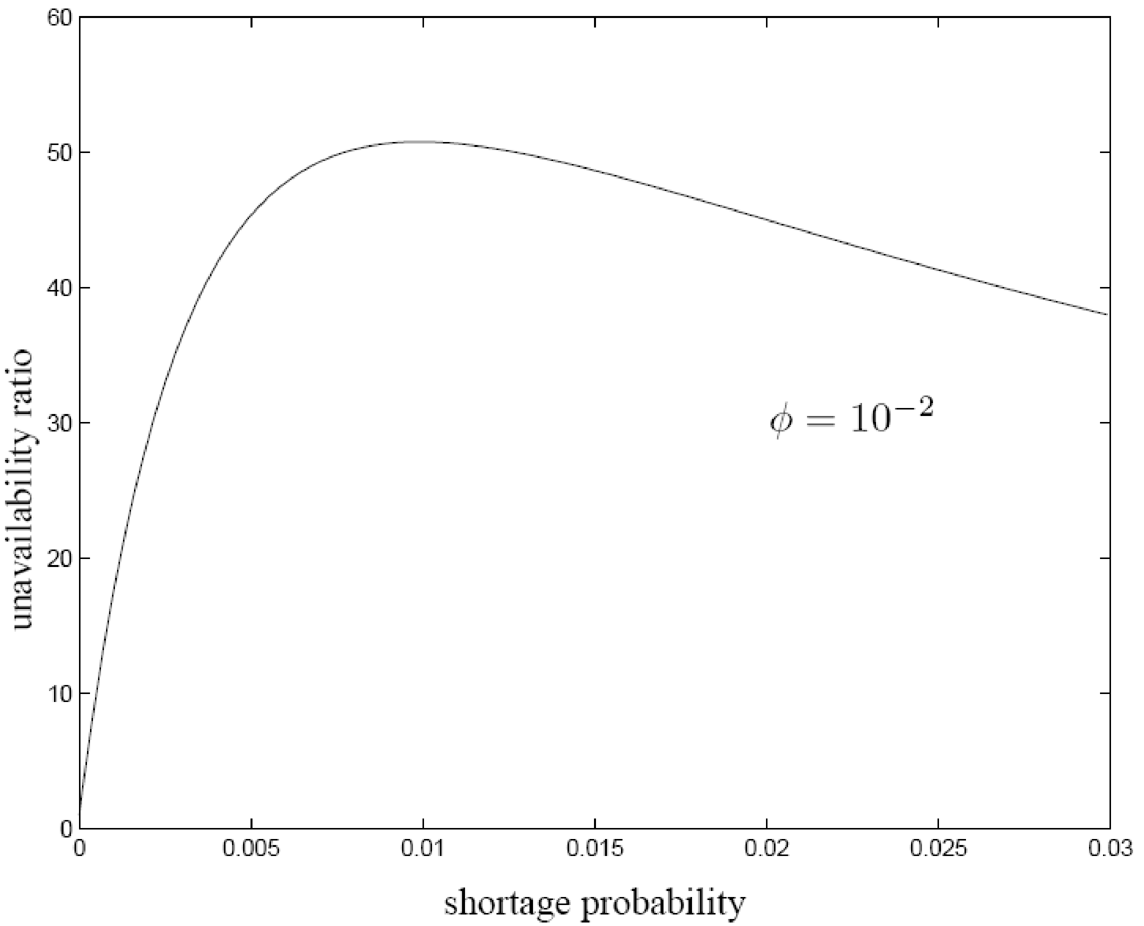
$\rho = 10^{-3}$



Numerical Examples

Unavailability ratio

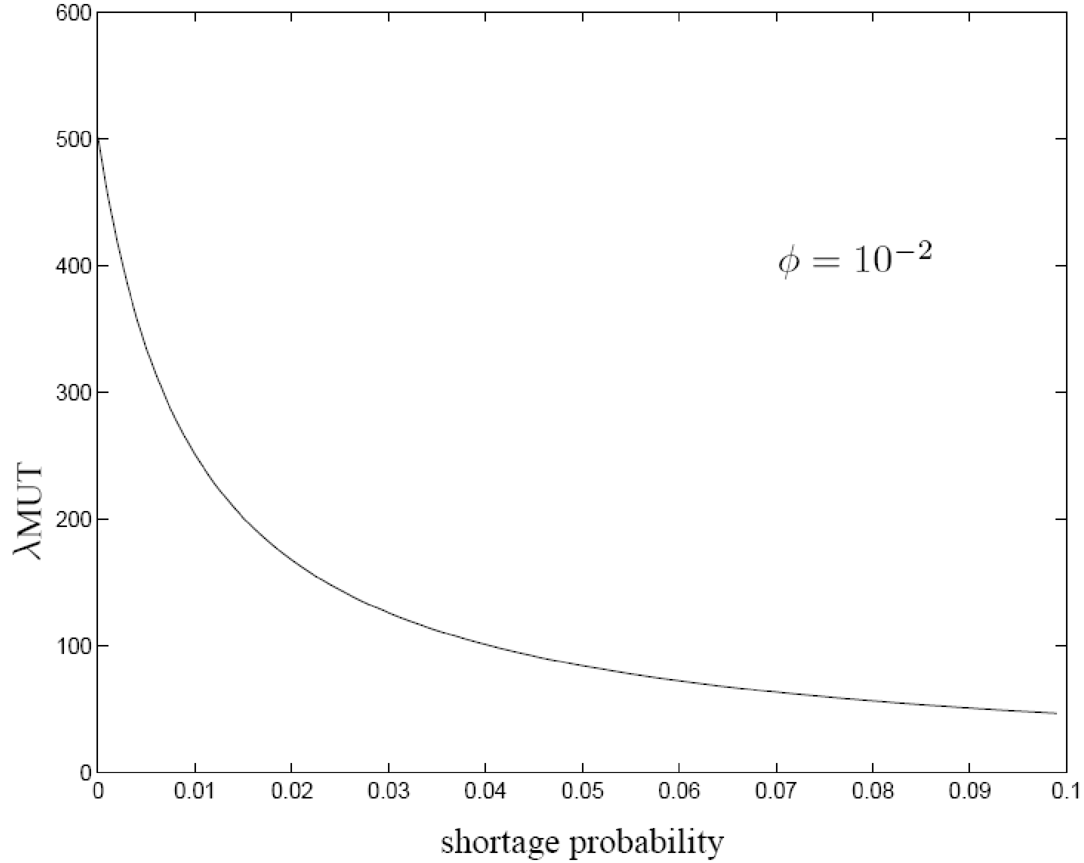
$\rho = 10^{-3}$



Numerical Examples

λMUT

$\rho = 10^{-3}$



Numerical Examples

Same result as with the heuristic

Therefore, λMUT ratio=1 !

Is λMDT ratio=1?

No since the heuristic gives :

$$2\mu MDT = 1 + \frac{\alpha}{\phi}$$

while our model gives :

$$2\mu MDT = 1 + \frac{\alpha}{\phi} \times \left(\frac{1 + 2/\phi}{1 + \alpha/\phi} \right)$$

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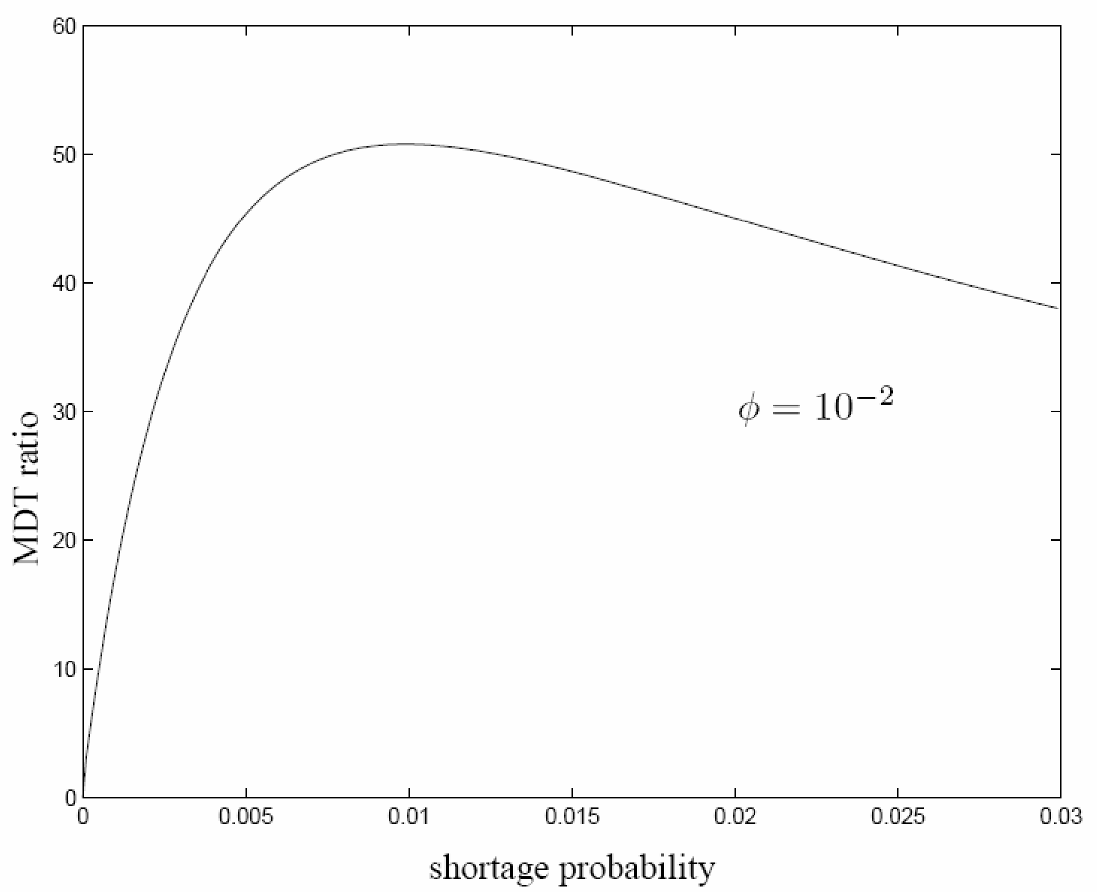
Numerical Examples

λMDT ratio

$\rho = 10^{-3}$



A more sophisticated model

In the previous model, we did not consider the maximum number of parts (which depends on the inventory policy).

Let us assume know that there is some kind of “kanban” policy such that the sum of the number of individual orders plus the number of parts in the workshop is equal to N.

➡ It is possible to consider the following Markovian model

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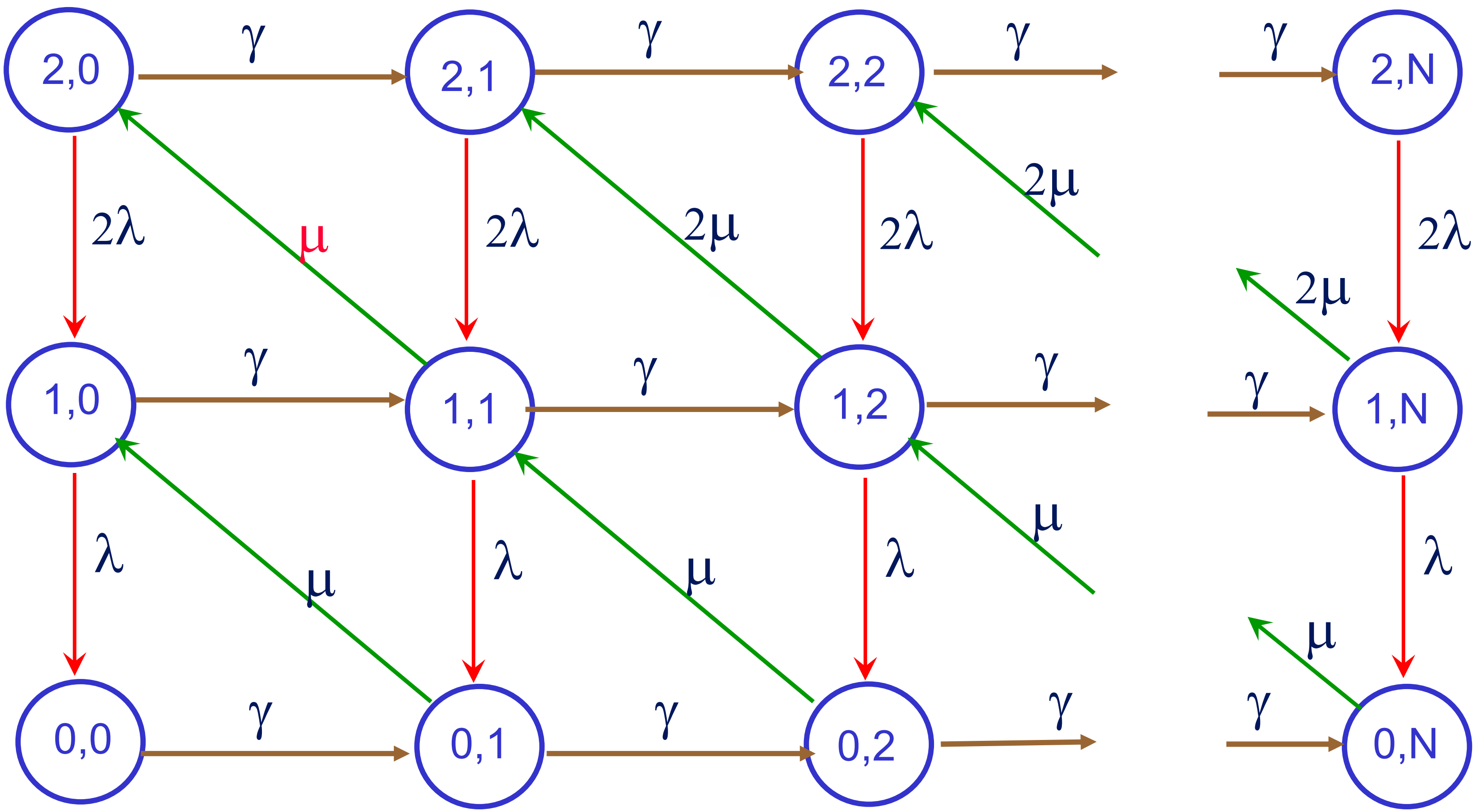
A more sophisticated model



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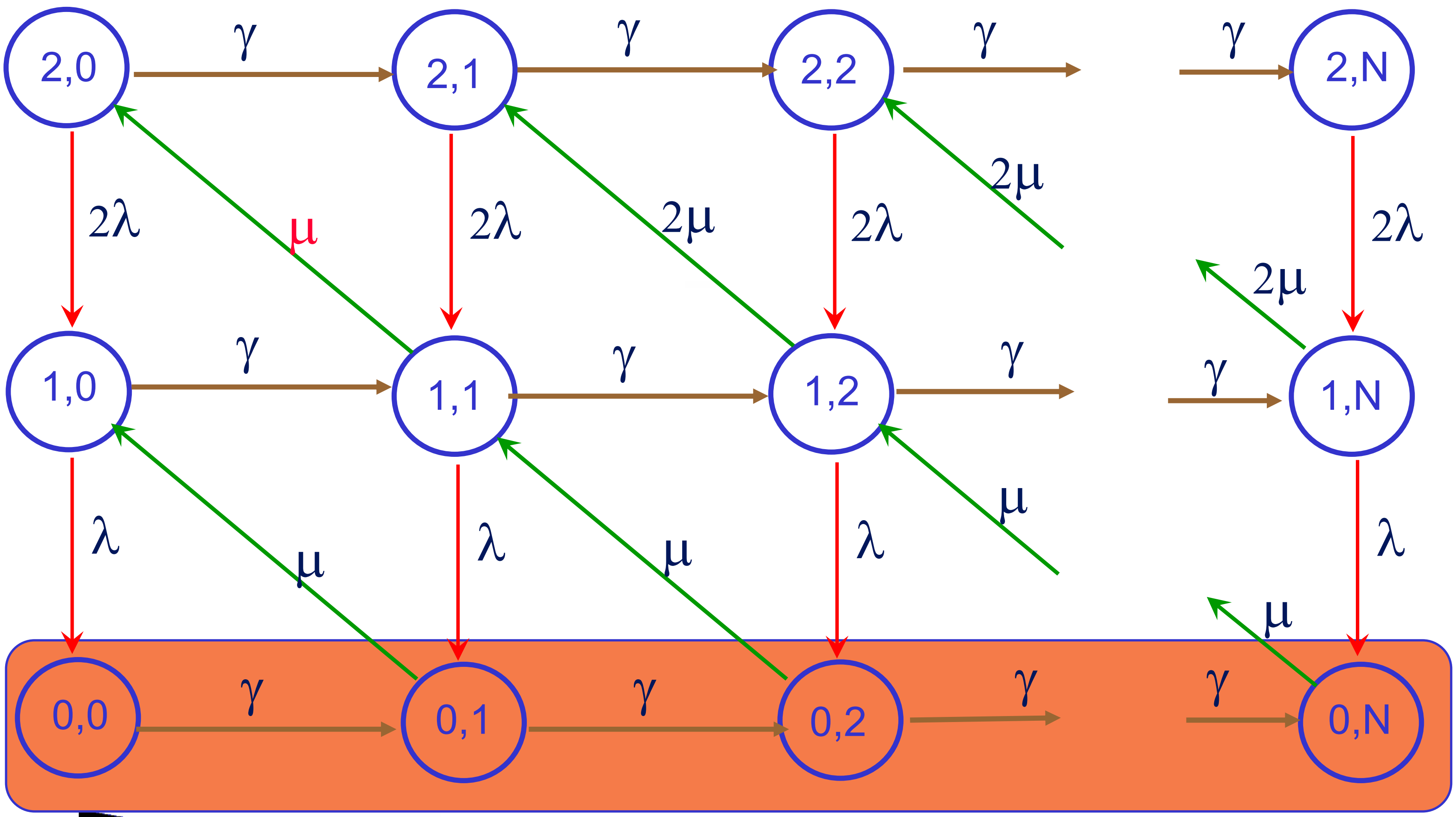
A more sophisticated model

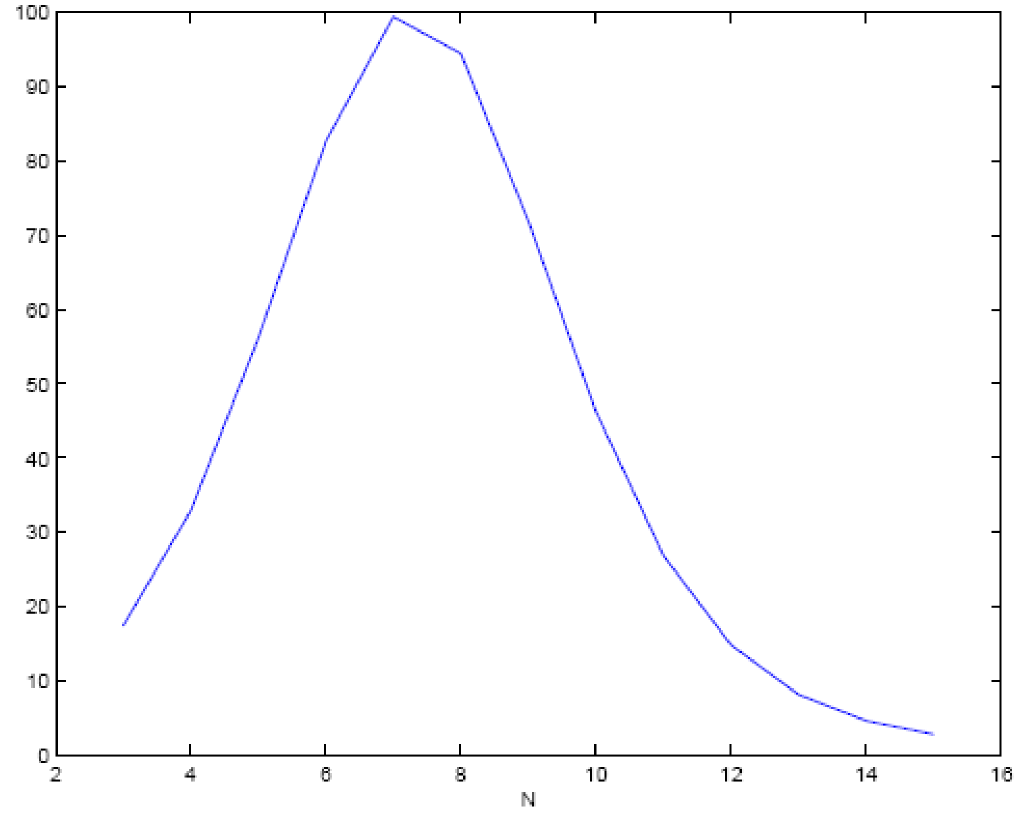


A more sophisticated model

Unavailability ratio

$\rho = 10^{-3}$ $\phi = 4.10^{-2}$

Numerical solution



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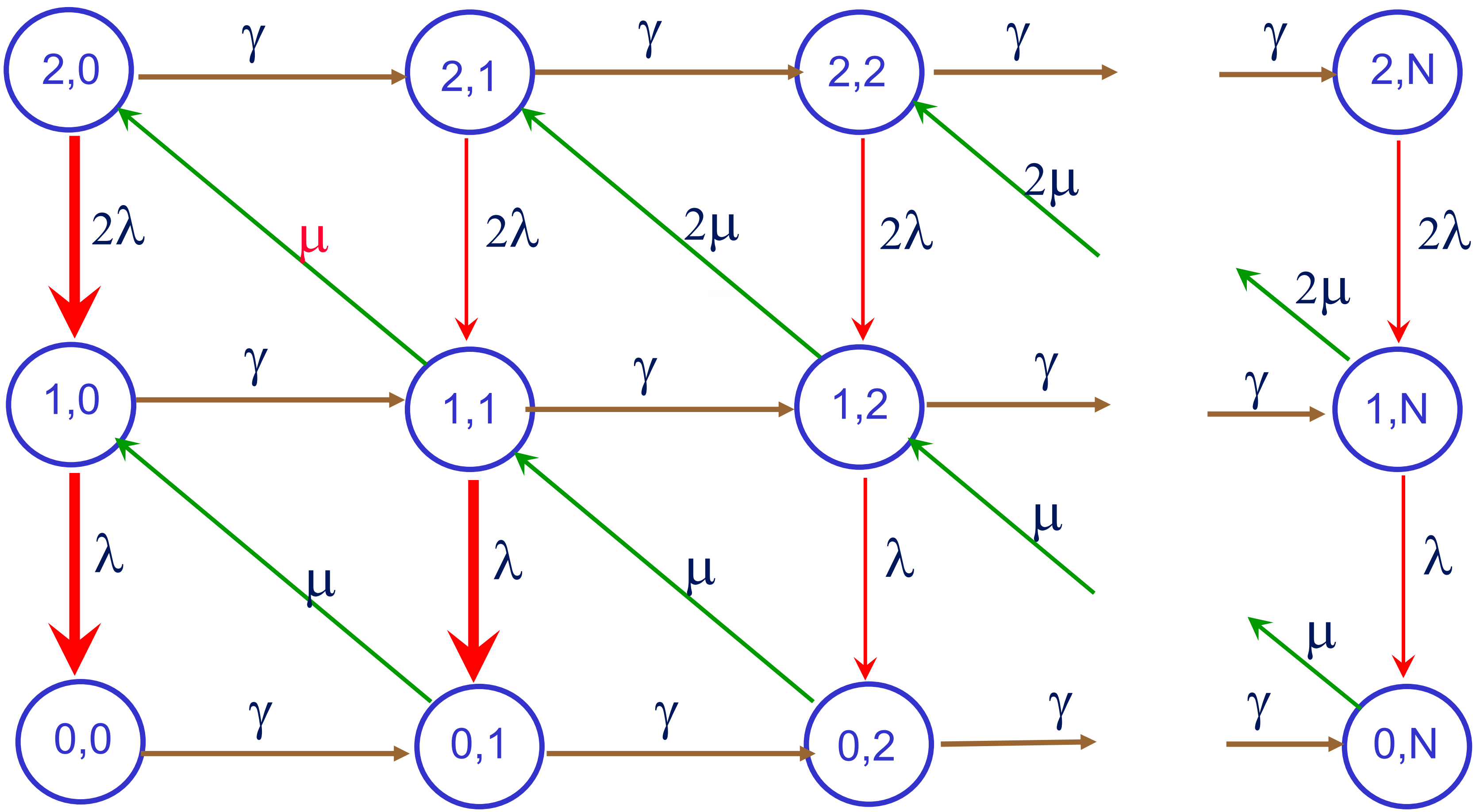
A more sophisticated model



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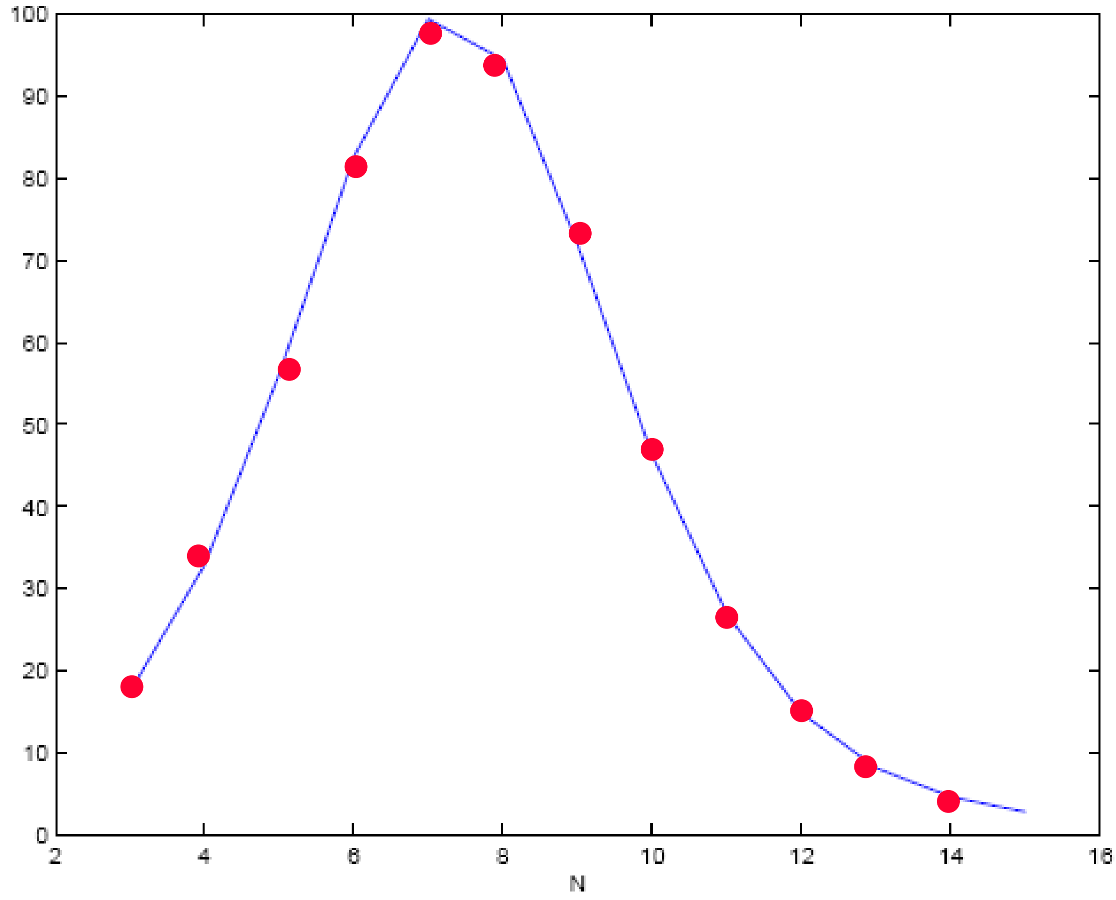
A more sophisticated model

Unavailability ratio

$\rho = 10^{-3} \quad \phi = 4.10^{-3}$

● : 6 states model



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Conclusions

In the situation of complex optimization problems for large redundant structures associated with their maintenance policies, we pointed out the risk of making small misunderstanding in the stage of modeling involving large errors on the evaluation of performance measures.

So, where is the risk ?

We could consider some extensions :

- new inventory policies
- larger redundant structures

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Thank You !